Calculus Through a Data Lens: Broadening Scope Through Data and Modeling

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Goucher College

- Approximately 1400 undergraduate, (mostly) residential students
- Located in Towson, Maryland (Baltimore County); just a few miles from Baltimore city
- Interesting facts: required study abroad, former women's college

Credit to Rob Ferrell. Courtesy of Goucher College.

Goucher's New Curriculum

Curriculum Goal

"Students will gain an integrative learning experience and develop proficiency in areas that will make them ready for the jobs of the future. "

Focus on acquiring skills applied across workplaces:

- Working with others different from oneself
- Analyzing and interpreting data
- Writing proficiency
- Global perspective

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Data Analytics

Two semesters of data analytics required for each student:

- Data Analytics Foundational (DAF) (75% of course)
  - Goal 1: Obtain and analyze Data
  - Goal 2: Draw and critique conclusions from data analysis
  - Goal 3: Communicate information from data analysis
- Data Analytics Across the Curriculum (DA-AC) (33% of course)
  - Build on DA-F skills to meet the three goals above in a discipline-specific way

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Goals

- Single <u>and</u> multivariable
- Application-driven
- Emphasis on interpreting results
- Data-driven
- Proficiency in computer software

Course Overview

- Originally (Fall 2017) adapted from MATH 135 at Macalester College (thanks for the help!)
- Precalculus review (ALEKS), daily homework (WeBWork), weekly quizzes, three tests, unit projects (RStudio), final exam
    - Unit 1 - Functions as Models
    - Unit 2 - Units, Dimension, and Estimation
    - Unit 3 - Concepts of Derivatives
    - Unit 4 - Symbolic Differentiation
    - Unit 5 - Optimization
    - Unit 6 - Integration and Accumulation
    - Unit 7 - Models of Change

The Text
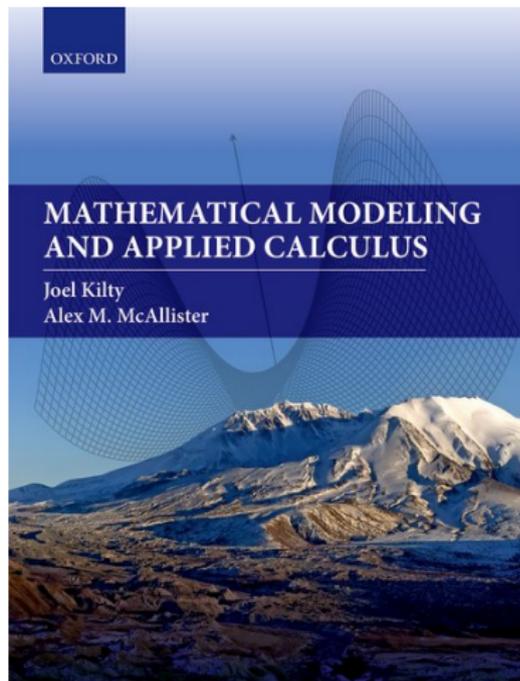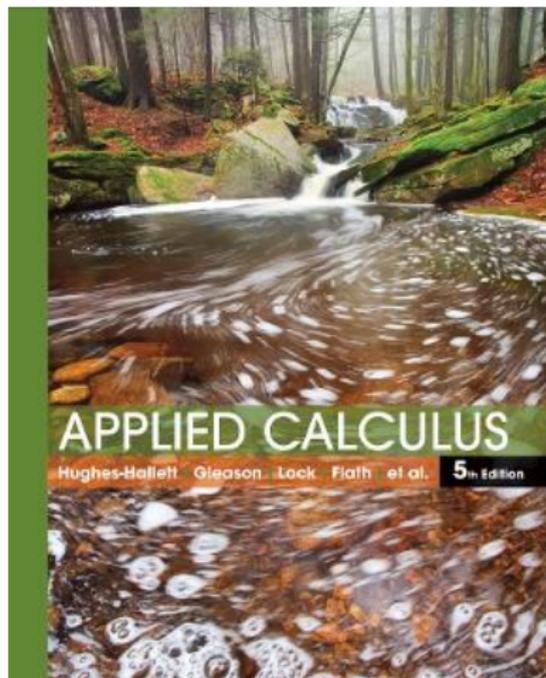
A New Major

- 2018 Program Prioritization: mathematics major/minor discontinued
- Ongoing discussions of creating new major
- Quick push to create new major: Integrative Data Analytics
- Major approved spring 2019, inaugurated fall 2019; 7 majors as of December
- Update needed: how does calculus serve *this* major?

Latest Iteration - Fall 2019

Weekly homework (textbook), weekly quizzes, two/three tests, unit projects (Excel), final exam

- Unit 1 - Fundamentals (Precalculus, Excel, Functions)
- Unit 2 - Concepts of Derivatives*
- Unit 3 - Symbolic Differentiation
- Unit 4 - Optimization
- Unit 5 - Integration and Accumulation**
- Unit 6 - Probability
- Unit 7 - Models of Change (Differential Equations)

* Add limits
** No multivariable integrals. Dropped unit conversion/dimensional analysis (explicitly)

Final Exam Question: The table below gives the number of households, $h(t)$, in millions, in the US with cable television t years since 1998.

t	0	2	4	6	8	10	12
$h(t)$	64.65	66.25	66.732	65.727	65.141	64.874	60.958

a. Estimate the value of $h'(4)$.

b. Explain what the value and the sign (positive or negative) of $h'(4)$ is telling you in terms of cable television.

Communicating in Context Cont.

Final Exam Question: The contour plot below describes corn production, $C = f(R, T)$ in percentage of the present production as a function of rainfall, R, in inches and temperature, T, in °F.
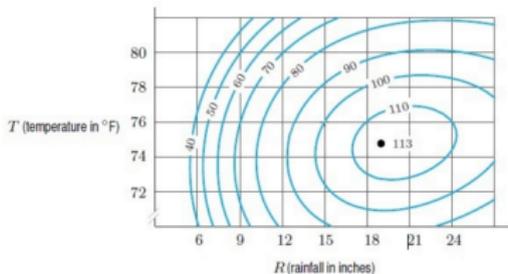
Estimate each of the given quantities, clearly showing your work. Give the proper units and write a complete sentence **interpreting your answers** in terms of corn production.

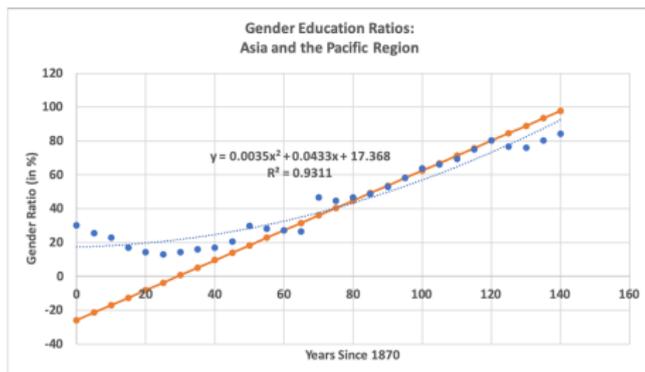a. $f_R(15, 76) = \left.\dfrac{\partial f}{\partial R}\right|_{(x,y)=(15,76)}$.

b. $f_T(15, 76) = \left.\dfrac{\partial f}{\partial T}\right|_{(x,y)=(15,76)}$.

Data Projects

Objective:

The projects in this course are designed to allow you to use the tools of calculus you have learned so far to analyze a dataset and draw conclusions. The projects are a chance for you to:

- Apply your mathematical tools in a real world context
- Learn to work on a team, building off each other's strengths
- Become proficient with using software to analyze and manipulate data and
- Develop skills in communicating your results in context (that is, without mathematical jargon).

Gender Education Ratios:
Asia and the Pacific Region

$y = 0.0035x^2 + 0.0433x + 17.368$
$R^2 = 0.9311$

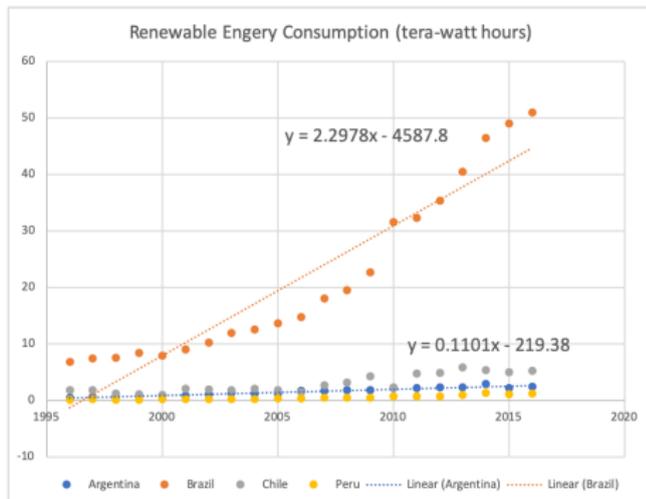Gender Ratio (in %)

Years Since 1870

1. First, approximate the instantaneous rate of change as the mean of the average rates of change between 1990 and 1995, and 1985 and 1990. Next, use the curve of best fit to approximate the instantaneous rate of change in 1990.

2. These two values should be different. Why do these methods produce different answers? Which one do you think is a better representation of the true instantaneous rate of change in 1990? Give reasons, both mathematical and historical, to support you answer.
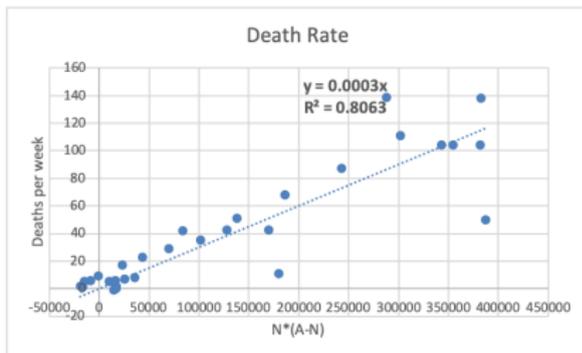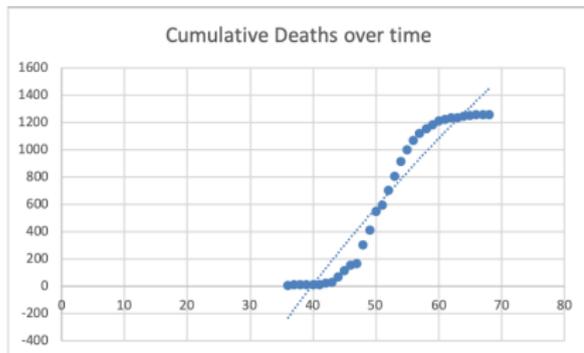
Project 2: Linear Regression

Year	Argentina	Brazil	Chile	Peru
1996	0.44048008	6.75270963	1.738	0.137
1997	0.58864567	7.378951	1.743	0.154
1998	0.59863538	7.50523301	1.161	0.126
1999	0.80519111	8.37600228	1.01	0.139
2000	0.83571644	7.855363	0.941	0.158
2001	0.75633184	8.990407	2.068	0.163
2002	1.06716899	10.226147	1.944	0.186
2003	1.29346736	11.894463	1.807	0.196
2004	1.1297911	12.4750557	2.031	0.153
2005	1.16276848	13.5909393	1.79	0.348
2006	1.7082772	14.768926	1.431	0.348
2007	1.67149248	18.0075085	2.696	0.417
2008	1.80180867	19.5236855	3.083	0.46
2009	1.8344685	22.639248	4.274	0.46
2010	2.12749301	31.5452401	2.247	0.673
2011	2.16711861	32.2346218	4.673	0.675
2012	2.23055045	35.2955788	4.854	0.668
2013	2.32452821	40.4713749	5.761	0.96
2014	2.84083911	46.3844257	5.327	1.291
2015	2.15042153	49.027348	4.99536747	1.07398747
2016	2.34269158	50.9442813	5.2542544	1.21618893

Regression Formulas	Argentina	Brazil	Chile	Peru
m	0.110118291	2.29784333	0.23265565	0.05608023
b	-219.3792966	-4587.7648	-463.81063	-112.02065

Renewable Engery Consumption (tera-watt hours)

$y = 2.2978x - 4587.8$

$y = 0.1101x - 219.38$

● Argentina ● Brazil ● Chile ● Peru ⋯⋯ Linear (Argentina) ⋯⋯ Linear (Brazil)

Project 3:
Modeling the 2009 H1N1 Outbreak

Cumulative Deaths over time — Death Rate

$y = 0.0003x$
$R^2 = 0.8063$

- Use derivatives to show when the number of deaths starts to decrease.
- Derive a differential equation to describe the death rate based on your regression line.

Multivariable Project Example: Estimating Accumulated Rainfall in Puerto Rico
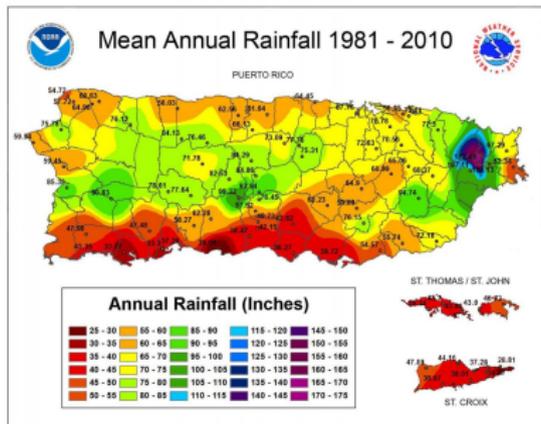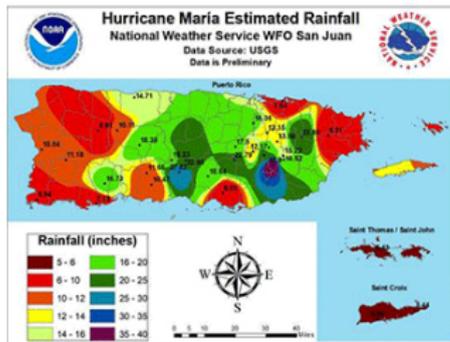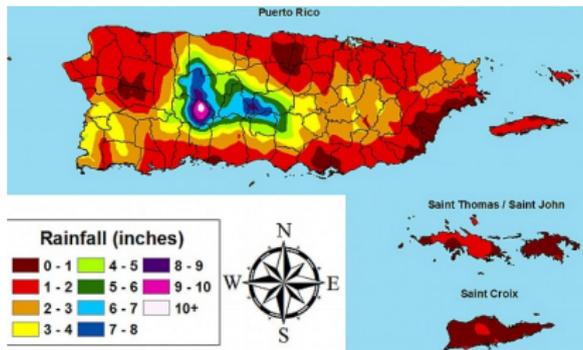
Student Feedback

Course Evaluation

"This whole class was really conceptually difficult for me. I have taken calculus previously but never through an analytic lens. Dr. G encouraged me to never get discouraged and to keep trying to see questions from different angles in order to better understand them."

Project Evaluation

"I enjoyed looking at numbers from outside sources and trying to apply reasons of why the numbers were the way they were.... I really enjoyed learning how to apply the math formulas and learnings to understand trends in the real world, and it was very rewarding and felt very accomplished once I was able to clearly connect the math to the real world examples."

Calculus has been a very difficult topic to learn for me, yet I have been doing really well in this course. I have realized that I am more of a theory-in-practice learner, rather than just a theory learner. This has made this class way more interesting and enjoyable. The most interesting thing I have learned is that there are applications to calculus for multi-variable situations. I think that seeing how this material is applicable to real-life situations make this class way more interesting and has made me perform well.

Next Steps

- Possible adoption of another book (and switch back to R?)
- Rework projects to make them more relevant, meaningful, and collaborative
- Continued re-evaluation of topics as IDA major grows

Thank you for coming! Feel free to reach out with questions/comments:
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